Probabilidades

$$\mu_Y = E[Y] = \sum_i y_i P(Y = y_i) \qquad \sigma_Y^2 = Var[Y] = \sum_i (y_i - \mu_y)^2 P(Y = y_i)$$

$$\mu_Y = E[Y] = \int y f_Y(y) dy \qquad \sigma_Y^2 = Var[Y] = \int (y - \mu_y)^2 f_Y(y)$$

$$E_Y[Y] = E_X[E_{Y|X}[Y|X]] \qquad E_Y[Y] = E_X[V_{Y|X}[Y|X]] + V_X[E_{Y|X}[Y|X]]$$

$$T = g(T) \Rightarrow f_T(t) = f_Y(g^{-1}(y)) \left| \frac{\mathrm{d}}{\mathrm{d}t} g^{-1}(t) \right|$$

Distribuições de Probabilidade

Distribuição	Probab./Densidade	Domínio	E[Y]	Var[Y]
$Y \sim \mathrm{U}(k)$	$\frac{1}{k}$	$y = 1, 2, \ldots, k$	$\frac{min(Y)+max(Y)}{2}$	$\sqrt{\frac{(max(Y)-min(Y)+1)^2-1}{12}}$
$Y \sim \mathrm{B}(n,p)$	$\binom{n}{y} p^y (1-p)^{n-y}$	$y = 0, 1, 2, \ldots, n$	np	$np(1-p)$
$Y \sim \mathrm{HG}(N,K,n)$	$\frac{\binom{K}{y}\binom{N-K}{n-y}}{\binom{N}{n}}$	$y = 0, 1, \ldots, \min(K,n)$	np	$np(1-p)\frac{N-n}{N-1}$
$Y \sim \mathrm{P}(\lambda)$	$\frac{e^{-\lambda}\lambda^y}{y!}$	$y = 0, 1, 2, \ldots$	λ	λ
$Y \sim \mathrm{G}(p)$	$(1-p)^y p$	$y = 0, 1, 2, \ldots$	$\frac{1-p}{p}$	$\frac{(1-p)}{p^2}$
$Y \sim \mathrm{BN}(r,p)$	$\binom{y+r-1}{r-1}(1-p)^y p^r$	$y = 0, 1, 2, \ldots$	$\frac{r(1-p)}{p}$	$\frac{r(1-p)}{p^2}$
$Y \sim \mathrm{U}[a,b]$	$\frac{1}{b-a}$	$a \le y \le b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
$Y \sim \mathrm{N}(\mu,\sigma^2)$	$\frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\frac{1}{2\sigma^2}(y-\mu)^2\}$	$y \in (-\infty, \infty)$	μ	σ^2
$Y \sim \mathrm{HN}(0,\sigma^2)$	$\frac{\sqrt{2}}{\sigma\sqrt{\pi}} \exp\{-\frac{y^2}{2\sigma^2}\}$	$y \in (0, \infty)$	0	σ^2
$Y \sim \mathrm{LN}(\mu,\sigma^2)$	$\frac{1}{y\sqrt{2\pi\sigma^2}} \exp\{-\frac{1}{2\sigma^2}(\ln(y)-\mu)^2\}$	$y \in (-\infty, \infty)$	$\exp\{\mu + \sigma^2/2\}$	$\exp\{2\mu + \sigma^2\}(\exp\{\sigma^2\} - 1)$
$Y \sim \mathrm{Exp}(\lambda)$	$\lambda \exp(-\lambda y)$	$y \ge 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
$Y \sim \mathrm{Exp}(\phi)$	$\frac{1}{\phi} \exp(-y/\phi)$	$y \ge 0$	ϕ	ϕ^2
$Y \sim \mathrm{Erlang}(\lambda, r)$	$\frac{\lambda^r}{(r-1)!} y^{r-1} \exp(-\lambda y)$	$y \ge 0, r = 1, 2, \ldots$	$\frac{r}{\lambda}$	$\frac{r}{\lambda^2}$
$Y \sim \mathrm{Ga}(p,q)$	$\frac{q^p}{\Gamma(p)}\ y^{p-1}\ \exp\{-qy\}$	$y \ge 0$	$\frac{p}{q}$	$\frac{p}{q^2}$
$Y \sim \mathrm{Ga}(\alpha,\beta)$	$\frac{1}{\beta^\alpha \Gamma(\alpha)}\ y^{\alpha-1}\ \exp\{-y/\beta\}$	$y \ge 0$	$\alpha\beta$	$\alpha\beta^2$
$Y \sim \chi^2(\nu)$	$\frac{1}{2^{\nu/2}\Gamma(\nu/2)\,\beta^\alpha}\ y^{\nu/2-1}\ \exp\{-y/2\}$	$y \ge 0$	ν	2ν
$Y \sim I\chi^2(\nu)$	$\frac{2^{\nu/2}}{\Gamma(\nu/2)\,\beta^\alpha}\ y^{-\nu/2-1}\ \exp\{-1/2y\}$	$y \ge 0$	$\frac{1}{\nu-2}$	$\frac{2}{(\nu-1)^2(\nu-4)}$
$Y \sim I\chi^2_{sc}(\nu,\varphi)$	$\frac{(\nu/2)^{\nu/2}\varphi^{\nu/2}}{\Gamma(\nu/2)} y^{-\frac{\nu}{2}-1} e^{-\nu\varphi/(2y)}$	$y \ge 0$	$\frac{\nu\varphi}{\nu-2}$	$\frac{2\nu^2\varphi^2}{[(\nu-2)^2(\nu-4)]}$
$Y \sim \mathrm{IGa}(p,q)$	$\frac{q^p}{\Gamma(p)}\ y^{-p-1}\ \exp\{-q/y)\}$	$y \ge 0$	$\frac{q}{p-1}$	$\frac{q^2}{(p-1)^2(p-2)}$
$Y \sim \mathrm{IGa}(\alpha,\beta)$	$\frac{1}{\beta^\alpha\Gamma(\alpha)}\ y^{-\alpha-1}\ \exp\{-1/(\beta y)\}$	$y \ge 0$	$1/(\beta(\alpha-1))$	$\frac{1}{\beta^2(\alpha-1)^2(\alpha-2)}$
$Y \sim \mathrm{UniGa}(\alpha,\beta)$	$\frac{\alpha^\beta}{\Gamma(\beta)} y^{\alpha-1}\left[\log\left(\frac{1}{y}\right)\right]^{\beta-1}$	$y \in (0,1)$	$\left(\frac{\alpha}{\alpha+1}\right)^\beta$	$\left[\left(\frac{\alpha}{\alpha+2}\right)^\beta - \left(\frac{\alpha}{\alpha+1}\right)^{2\beta}\right]$
$Y \sim \mathrm{Weibull}(\alpha,\beta)$	$\frac{\alpha}{\beta}\ (y/\beta)^{\alpha-1}\exp\{-(y/\beta)^\alpha\}$	$y \ge 0$	$\beta\,\Gamma(1+\frac{1}{\alpha})$	$\beta^2\left[\Gamma(1+\frac{2}{\alpha}) - \Gamma^2(1+\frac{1}{\alpha})\right]$
$Y \sim \mathrm{Beta}(\alpha,\beta)$	$\frac{1}{B(\alpha,\beta)}\ y^{\alpha-1}(1-y)^{\beta-1}$	$0 \le y \le 1$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\ \beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
$Y \sim \mathrm{BetaG}(\alpha,\beta,a,b)$	$\frac{(b-a)^{-(\alpha+\beta-1)}}{B(\alpha,\beta)}(t-a)^{\alpha-1}(b-t)^{\beta-1}$	$a \le y \le b$	$\frac{\alpha b+\beta a}{\alpha+\beta}$	$\frac{(\alpha b+\beta a)-(c+d)}{\alpha+\beta-2}$
$Y \sim \mathrm{Pareto}(\alpha, y_m)$	$\frac{\alpha y_m^\alpha}{y^{\alpha+1}}$	$y \ge y_m$	$\frac{\alpha y_m}{\alpha-1}$	$\frac{\alpha y_m^2}{(\alpha-1)^2(\alpha-2)}$
$Y \sim \mathrm{Lomax}(\alpha,\beta)$	$\frac{\alpha}{\beta}\left[1+\frac{y}{\beta}\right]^{-(\alpha+1)} = \alpha\frac{\beta^\alpha}{(\beta+y)^{\alpha+1}}$	$t \ge 0$	$\frac{\beta}{\alpha-1}$	$\frac{\alpha\beta^2}{(\alpha-1)^2(\alpha-2)}$
$Y \sim \mathrm{Logistica}(\mu,\lambda)$	$\frac{e^{-(y-\mu)/\lambda}}{\lambda\left(1+e^{-(y-\mu)/\lambda}\right)^2}$	$y \in (-\infty, \infty)$	μ	$\frac{\lambda^2\pi^2}{3}$

$$Z = (Y-\mu)/\sigma \qquad \Gamma(\alpha) = \int_0^\infty y^{\alpha-1}\ \exp\{-y\}\ dy \qquad \Gamma(\alpha) = (\alpha-1)! \qquad \Gamma(\alpha+1) = \alpha\,\Gamma(\alpha) \qquad B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$